

A new "polarized version" of the Casimir Effect is measurable

O. Kenneth and S. Nussinov

*School of Physics and Astronomy
Beverly and Raymond Sackler Faculty of Exact Sciences
Tel Aviv University, Ramat Aviv, 69978, Israel*

Abstract

We argue that the exactly computable, angle dependent, Casimir force between parallel plates with different directions of conductivity can be measured.

Some time ago we presented [N] heuristic arguments motivated by the "vacuum pressure" picture[Mi] for the Casimir force[C]. These arguments suggested that the attraction between parallel plates conducting along different directions, decreases monotonically as a function of γ the angle between these directions. This angle dependent effect was then evaluated exactly[KN1,KT] as part of the thesis work of O.Kenneth. The (euclidean) path integral technique¹ used for this purpose was usefull for computing and understanding a large variety of Casimir related issues[KT,KN2]. Our main purpose here is to note that the angle dependent Casimir effect (which is present in the electromagnetic case because of the polarization degree of freedom) is not only exactly computable. Rather we argue that it is also measurable with a precision of few percent-the precision of recent experiments[L,M] of the ordinary Casimir effect between (isotropically) conducting parallel plates.

Let us briefly recall the derivation of the angle dependent Casimir effect. The euclidean² partition function in the presence of two disjoint conducting surfaces Σ_1 and Σ_2 can be written as:

$$\int \mathcal{D}J_{(1)}^\mu(x) \mathcal{D}J_{(2)}^\mu(y) \exp \left\{ - \int J_{(1)}^\mu(x) \frac{dx dx'}{(x-x')^2} J_{(1)}^\mu(x') - 2 \int J_{(1)}^\mu(x) \frac{dx dy}{(x-y)^2} J_{(2)}^\mu(y) - \int J_{(2)}^\mu(y) \frac{dy dy'}{(y-y')^2} J_{(2)}^\mu(y') \right\} \quad (1)$$

In the above $x, x' \in \tilde{\Sigma}_1$ and $y, y' \in \tilde{\Sigma}_2$ with $\tilde{\Sigma}_i = \Sigma_i \otimes \text{time axis}$. This

expression is obtained by starting with the Maxwell (euclidean) action $\int F_{\mu\nu}^2$, using

$$\int_{x \in \tilde{\Sigma}_1} A_\mu(x) J_{(1)}^\mu d\sigma_1 + \int_{x \in \tilde{\Sigma}_2} A_\mu(x) J_{(2)}^\mu d\sigma_2$$

¹These techniques were introduced some time ago and extensively used by Kardar and collaborators (K1)(K2). The specific application for the new angle dependent effect (KN1) to the effect for general planar geometry(K), and to motivate the attraction between disjoint objects of similar ϵ/μ ratios (KN2)are however new .

²The Casimir problem for two conducting surfaces Σ_1 and Σ_2 is fully defined by space like vectors only:the vectors x_i and y_j connecting points on the two conductors to their respective centroids, and a the relative displacement of the latter. Hence there can be no obstruction to complete Wick rotation and formulation of the problem via euclidean action and partition function.

with the (conserved) currents $J_{(1)}^\mu, J_{(2)}^\mu$ serving as lagrange multipliers forcing the boundary conditions $E_{\parallel} = 0$ on Σ_1, Σ_2 respectively, doing the gaussian $\mathcal{D}A_\mu$ integration, and utilizing $\partial_\mu J^\mu = 0$ to justify the choice $\Delta^{\mu\nu}(x - y) = \frac{g^{\mu\nu}}{(x-y)^2}$ (see ref[KT,KN1] for details). The $(x - y)^{-2}$ coefficients in the quadratic form in the currents in eq(1) above is clearly the source of divergences in Casimir energy evaluation. These divergences arise however **only** from products of two $J^{(1)}$'s at near by points on Σ_1 or of two $J^{(2)}$'s at near by points on Σ_2 . Let us divide out $\mathcal{Z}(a)$ by $\mathcal{Z}(a \rightarrow \infty)$ which from the definition of $\mathcal{Z} = e^{-E(a)T}$ with T the size of the time interval is equivalent to subtracting from the total Casimir energy of the system the separate energies of the two conductors Σ_1 and Σ_2 . The infinite contributions $\frac{\lim_{x_i \rightarrow x_j} J_i^{(1)} J_j^{(1)}}{(x_i - x_j)^2}$ clearly divide out for each point $x_i \in \Sigma_1$ etc. Hence no divergences are expected³ and

$$\frac{e^{-E(a)T}}{e^{-E(\infty)T}} = \frac{\int \prod dJ^{(1)}(x) \prod dJ^{(2)}(y) e^{-\int \frac{J^{(1)}(x)J^{(1)}(x')}{(x-x')^2}} e^{-\int \frac{J^{(1)}(x)J^{(2)}(y)}{(x-y)^2}} e^{-\int \frac{J^{(2)}(y)J^{(2)}(y')}{(y-y')^2}}}{\int \prod dJ^{(1)}(x) \prod dJ^{(2)}(y) e^{-\int \frac{J^{(1)}(x)J^{(1)}(x')}{(x-x')^2}} e^{-\int \frac{J^{(2)}(y)J^{(2)}(y')}{(y-y')^2}}} \quad (2)$$

is finite and well defined (the coefficient of the mixed $J^{(1)}(x)J^{(2)}(y)$ product $\frac{1}{(x-y)^2}$ is bound by $\frac{1}{a^2}$, with $a = \min\{|x - y|; x \in \Sigma_1, y \in \Sigma_2\}$ the minimal distance between the conductors and is finite) The expression in eq(2) can serve as a meaningful starting point for numerical estimates or general considerations[KN2]. The latter strongly suggest that $E(a)$ is monotonic and hence that the Casimir force between two disjoint conductors is attractive at all distances. For the special case at hand Σ_1 and Σ_2 are two infinite plates parallel to the x, y plane and separated by $|\vec{a}| = |a\hat{z}| = a$. The general expression of eq (2) becomes:

³This is in clear contrast to the casimir energy of each of the two conductors separately. The latter diverge and require careful regularization. For the mixed products or the mutual casimir energy a serves as a regulator. While we are appealing to the lore of renormalization theory and identify all the divergent parts with the energies of the separate conductors, a rigorous proof that the $Z(a)/Z(\infty)$ ratio (or the $E(a) - E(\infty)$ is independent of the scheme of renormalization will not be supplied here.

$$\mathcal{Z}(a) = \int \mathcal{D}J \exp \left(-\frac{1}{8\pi^2} \int d^3x d^3y \left(\frac{\vec{J}_1(x) \cdot \vec{J}_1(y) + \vec{J}_2(x) \cdot \vec{J}_2(y)}{(x-y)^2} + \frac{2\vec{J}_1(x) \cdot \vec{J}_2(y)}{(x-y)^2 + a^2} \right) \right) \quad (3)$$

With $\vec{J}_{1,2}$ ordinary 3-vectors in the 3-dimensional euclidean space $\vec{x} = (x, y, t)$. Fourier transforming in \vec{x} we obtain:

$$\mathcal{Z} = \int \mathcal{D}J \exp \left(-\frac{1}{8\pi^2} 4\pi^2 \int d^3k \left(\frac{\vec{J}_1(\vec{k}) \cdot \vec{J}_1(-\vec{k}) + \vec{J}_2(\vec{k}) \cdot \vec{J}_2(-\vec{k})}{k} + \frac{2\vec{J}_1(\vec{k}) \cdot \vec{J}_2(-\vec{k})}{k} e^{-ka} \right) \right) \quad (4)$$

where $\vec{k} = (k_x, k_y, k_t)$, $k = |\vec{k}|$ and we used translation invariance and $\int d^3x e^{-ikx} \frac{1}{x^2+a^2} = \pi \frac{e^{-ka}}{k}$. In the usual case of two conducting parallel plates both $\vec{J}_1(k)$ and $\vec{J}_2(k)$ have two degrees of freedom corresponding to the two transverse directions which satisfy current conservation condition: $\vec{k} \cdot \vec{J} = 0$. In the case of specific conduction directions $\vec{J}_1(k)$ and likewise $\vec{J}_2(k)$ have only one allowed nonzero component determined by current conservation and by the demand that its spatial part (J_x, J_y) is along the direction of conduction. Let us denote the cosine of the angle between the directions of $\vec{J}_1(\vec{k})$ and $\vec{J}_2(\vec{k})$ by $\alpha(\hat{k})$. \vec{J}_1, \vec{J}_2 are vectors in the 3-dimensional Euclidean space (k_x, k_y, k_t) . Using ordinary geometry we find an explicit expression for $\alpha(k)$ in terms of the direction of \vec{k} and of the conduction directions in the two plates

$$\alpha^2 = \frac{[\cos \gamma - \sin^2 \theta \cos \varphi \cos(\varphi - \gamma)]^2}{(1 - \sin^2 \theta \cos^2 \varphi)(1 - \sin^2 \theta \cos^2(\varphi - \gamma))} \quad (5)$$

where $\vec{k} = (k_x, k_y, k_t) = (k \sin \theta \cos \varphi, k \sin \theta \sin \varphi, k \cos \theta)$ and γ is the angle between the directions of conductance in the two plates

Then we can write

$$\mathcal{Z} = \int \mathcal{D}J \exp \left(-\frac{1}{2} \int d^3k \left(\frac{J_1(k)J_1(-k) + J_2(k)J_2(-k)}{k} + \frac{2\alpha(k)J_1(k)J_2(-k)}{k} e^{-ka} \right) \right) \quad (6)$$

where the J 's appearing in the last equations are scalars. $J_1(k)^* = J_1(-k)$ and $J_2(k)^* = J_2(-k)$ forms the reality condition on J . Since the action is quadratic, \mathcal{Z} is given by the corresponding determinant which is just the

product of the two-dimensional determinants corresponding to the various value of \vec{k} . Hence

$$\frac{\mathcal{Z}(a)}{\mathcal{Z}(\infty)} = \prod_k \det \begin{pmatrix} \frac{1}{k} & \frac{\alpha(\hat{k})}{k} e^{-ka} \\ \frac{\alpha(\hat{k})}{k} e^{-ka} & \frac{1}{k} \end{pmatrix}^{\frac{1}{2}} / \prod_k \det \begin{pmatrix} \frac{1}{k} & 0 \\ 0 & \frac{1}{k} \end{pmatrix}^{\frac{1}{2}} \quad (7)$$

and

$$\begin{aligned} \ln \frac{\mathcal{Z}(a)}{\mathcal{Z}(\infty)} &= \frac{1}{2} \ln \det(\dots) - \frac{1}{2} \ln \det(\dots) \\ &= \frac{1}{2} AT \int \frac{d^3 k}{(2\pi)^3} \ln \left\{ \left| \begin{pmatrix} \frac{1}{k} & \frac{\alpha(\hat{k})}{k} e^{-ka} \\ \frac{\alpha(\hat{k})}{k} e^{-ka} & \frac{1}{k} \end{pmatrix} \right| - \ln \frac{1}{k^2} \right\} \\ &= \frac{1}{2} AT \int \frac{d^3 k}{(2\pi)^3} \ln(1 - \alpha(\hat{k})^2 e^{-2ka}) \end{aligned} \quad (8)$$

where the area-time AT came from density of states factor. It corresponds to having $\sum_k \rightarrow V \int \frac{d^3 k}{(2\pi)^3}$ the usual quantization of continuous modes in a box of volume V . Also note that a factor $\frac{1}{2}$ survives since we integrate over $dRe(J_1(k)), dIm(J_1(k))$ but only over half of the \vec{k} values say with $k_x > 0$. As expected the last integral is well defined and convergent.

Identifying $\ln \mathcal{Z} = ET$ we get finally:

$$E/A = \frac{1}{2} \int \frac{d^3 k}{(2\pi)^3} \ln(1 - \alpha(\hat{k})^2 e^{-2ka}) \quad (9)$$

using integration by parts this can (see ref[KN1] for more details) be written as

$$E/A = -\frac{1}{48a^3} \int \frac{d^3 k}{(2\pi)^3} \frac{k \alpha(\hat{k})^2}{e^k - \alpha(\hat{k})^2} \quad (10)$$

Using (5) our final result is⁴:

$$\begin{aligned} E/A &= -\frac{1}{48a^3} \frac{1}{(2\pi)^3} \int_0^\infty dk \int_0^{2\pi} d\varphi \int_0^\pi d\theta \times \\ &\times \left\{ \frac{k^3 \sin \theta (\cos \gamma - \sin^2 \theta \cos \varphi \cos(\varphi - \gamma))^2}{(1 - \sin^2 \theta \cos^2 \varphi) (1 - \sin^2 \theta \cos^2(\varphi - \gamma)) e^k - (\cos \gamma - \sin^2 \theta \cos \varphi \cos(\varphi - \gamma))^2} \right\} \end{aligned} \quad (11)$$

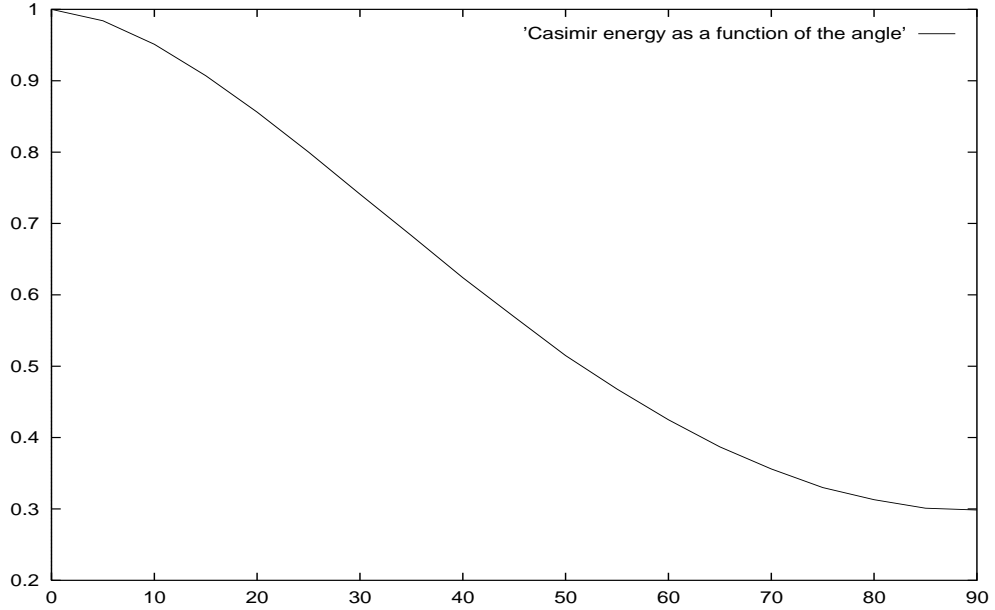
⁴It is possible to derive the angle dependent effect directly via mode summation analogous to the original casimir approach (K.T).

For $\gamma = 0$ we have $\alpha \equiv 1$ and $E/A = -\frac{1}{48a^3} \int \frac{4\pi}{(2\pi)^3} \frac{k^3 dk}{e^k - 1} = -\frac{1}{96a^3\pi^2} \sum_n \int_0^\infty k^3 e^{-nk} dk = -\frac{6 \sum \frac{1}{n^4}}{96a^3\pi^2} = \frac{-\frac{6\pi^4}{90}}{96a^3\pi^2} = -\frac{\pi^2}{1440a^3}$ The last expression is exactly half the ordinary Casimir energy for isotropic conductivity. The extra factor of two is expected since J_1 and J_2 (and the field they cause to vanish on Σ_1 and Σ_2) are scalar. The factor of two is due to the two polarizations in the usual sum over modes.

For general γ we use eq (11) and numerical integration gives the following:

γ	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°
E/E_0	1.000	0.984	0.951	0.907	0.856	0.800	0.741	0.683	0.624	0.569
γ	50°	55°	60°	65°	70°	75°	80°	85°	90°	
E/E_0	0.515	0.468	0.425	0.387	0.356	0.330	0.313	0.301	0.2984897749	

fig 1 represents the same $E(\gamma)/E_0$ data graphically



Naturally occurring materials conducting only along some preferred directions would have provided an ideal setting for testing the angle dependent effect. Indeed such materials would have manifested the unisotropic conductivity already on atomic Angstrom scales. We are not aware of any adequate candidates. ⁵ Fortunately unisotropic conductivity at atomic scales is not

⁵The materials exhibiting the high T_c phenomenon have a layered structure and tend

required for verifying our effect. The Casimir forces are intimately related to the Casimir Polder interactions. This implies a similar (retardation) lower bound for the interplate distance a , at which the Casimir force will not be masked by other effects:

$$a \geq a_{min} \approx \frac{d_{atom}}{(v_F/c)} \quad (12)$$

where d_{atom} is a relevant dimension, say the inter atomic distance in the crystal of order (few) Angstroms, and v_F is the Fermi velocity. For the conduction electron in gold the latter is of the order of 10^8 cm/sec, and hence $a_{min} \approx 0.1 \mu$. Indeed measurements of the ordinary Casimir effect were performed at yet larger distances: $0.5\mu < a < 5\mu$ and $0.2\mu < a < 2\mu$ in [9] and [10] respectively.

The interplate separation, a , sets the basic scale and cutoff for the Casimir problem. A fact, which as we will elaborate on next, is crucial for the proposed new experiments. Our suggestion is to obtain the unisotropic conductivity via “striped coating” by a conducting layer. The above suggests that taking the distance between the stripes, d , to be say $d \simeq a/2$ will suffice for the purpose of testing the new angle dependent effect. Indeed expanding equation (10) above we have:

$$\frac{E}{A} = \sum_n \int d^2\hat{k} \frac{[\alpha(\hat{k})]^{2n}}{n} \int_0^\infty dk k^2 e^{-2nka} \quad (13)$$

Integrating over k from 0 to ∞ yields

$$\frac{E}{A} = \sum_{n=1}^\infty d^2\hat{k} [\alpha(\hat{k})]^{2n} \frac{1}{n^4 8a^3}. \quad (14)$$

Both the n sum *and* the k integrations above are rapidly convergent. Specifically, the $n = 1$ term contributes at least $90/\pi^4 \approx 90\%$ of the total E/A . (This happens for $\gamma = 0$ and $\alpha = 1$. In general $\alpha < 1$ and the relative contribution of the $n > 1$ terms is further suppressed). The k integration

to conduct mainly in the planes of the layers. If we would cut such materials by a plane perpendicular to these layers then we would obtain a surface with a striped pattern of conduction. However, precisely because of the fact that one is not cutting along a natural cleavage direction the required smoothness of surfaces cannot be thus achieved.

is exponentially suppressed once $k > a^{-1}$. Since modes of wave number k can resolve surface details (or the induced currents $J_\mu(x)$) only at scales $\Delta x \sim k^{-1}$, the unisotropy of the conductivity needs to manifest only at a distance scale $\approx a$. This feature alongside the recently performed ordinary Casimir experiments motivates our suggested experiments for testing the new angle dependent effect.

To achieve the required smoothness of surfaces and sensitivity, and in order to avoid possible domination of the forces by random, nearby contact points, the recent (and older! [11]) Casimir experiments all share the following three elements:

1. “Perfect” quartz surfaces are coated by thin layers (of thickness $\approx 0.1 \mu$) of conducting metals (say gold) and thus form the required smooth conducting surfaces
2. Rather than having two parallel plates the experiments utilize one flat plate and a plano-convex spherical lens. The radius of curvature of the latter R is much larger than a the distance at the point of nearest proximity⁶
3. Like in all sensitive measurements of small forces, dating back to the Cavendish measurement of G_N , a torque balance is used.

Our suggested methodology parallels very much the above. The key new element is that the coating is done along parallel stripes in the plane of the plate, say along the x axis, with distance d between the centers of the stripes. More precisely we have a fraction f of the original quartz surface covered and a fraction $(1 - f)$ is left uncovered. The width of the conducting stripes (insulating intervening stripes) are then fd (and $(1 - f)d$), respectively. Even if d is 0.5μ and $f = 0.5$, present MBE (molecular beam epitaxy) and other (nano-) technologies enable us to generate the required uniform parallel stripes. Also this striped coating can be done over areas comparable with the “effective plate area” contributing in the above Casimir measurements. The angle dependent force $F/A \sim W(\gamma)/a^4$ allows a more detailed verification of the Casimir Phenomena.

To ensure its feasibility we have however to verify that the “Standard obstacles”, familiar from the ordinary Casimir experiments, and some new difficulties peculiar to the striped variant, can be avoided in the new context. The rest of this note is devoted to discussing these issues.

⁶Under these circumstances one can evaluate the casimir energy force by integrating over concentric annuli. This yields an effective plate area $A = \pi L^2$ with $L = \sqrt{2Ra}$

1)Finite conductivity corrections.

The ω dependence of ϵ and σ implies that the latter tend to vanish for frequencies higher than the plasma frequency $\omega_p = (4\pi ne^2/m_e)^{1/2}$. Modes with $\omega > \omega_p$ are effectively not reflected and do not contribute to Casimir effects. Unless this intrinsic cutoff is larger than $\frac{1}{a}$:

$$c/\omega_p = \frac{c}{(4\pi ne^2/m_e)^{1/2}} \leq a \quad (15)$$

appreciable finite conductivity corrections [12] and a reduction of the strength of the casimir effect cannot be avoided. Since the density of conduction electrons, n , and the other parameters in ω_p cannot be varied for bulk materials, eq. (15) can be viewed as yet another lower bound, in addition to the retardation bound of eq(15), on the plate separations for which the effect can be measured. If we use $n \simeq d_{atom}^{-3}$ for the density of (gold) atoms and corresponding conduction electrons and also estimate via the virial theorem $\langle m_e v^2 \rangle \simeq m_e v_F^2 \simeq \langle \frac{e^2}{r} \rangle \simeq \frac{e^2}{d_{at}}$, the limits (12) and (15) become the same, up to a numerical coefficient. The last bound becomes more stringent when only a fraction f of the, say, quartz surface is covered with a gold layer of thickness t . In this case an infalling electromagnetic plane wave which in bulk matter could have coherently interacted with all conduction electrons down to a depth of $l_{eff} \simeq 1/k \simeq c/\omega$, interacts only with a fraction $ft/l_{eff} = ft\omega/c$ thereof. The effective smeared electron density that should be substituted in this case for n in the expression for ω_p , should therefore be accordingly reduced: $n \rightarrow n' = ntf\omega/c$. In this case even lower frequencies, exceeding $\omega'_p = \sqrt{\frac{4\pi ne^2 ft\omega}{m_e c}}$ or equivalently:

$$\omega_{cutoffnew} = \frac{4\pi ne^2 ft}{m_e c} \quad (16)$$

will cease to contribute to the Casimir effect. The discussion following eq. 12 of the connection between the separation a and the relevant contributing modes with $k = \omega/c$ then implies the, new, finite conductivity corrected (F.N. 10), effective lower bound on the plate separation a :

$$a \geq \frac{\pi c}{\omega_{conew}} \simeq \frac{1}{4\lambda_e \alpha_{em} n t f} \quad (17)$$

(17) with $\alpha_{em} = 1/137$ and $\lambda_e = \hbar/(m_e c)$. The key point we would like to make is that also the last bound still allows our suggested measurements

down to plate separations of $a = 0.5\mu$. Thus for concreteness take the case of thickness $t = 0.01\mu$ gold coating of half the surface. In this case $n = \frac{\rho_{AU}}{A_{AU}} N_{av} \simeq 6.10^{22}$. and and eq. 16 reads

$$a/\mu \geq 0.3(f/(\frac{1}{2}))(t/0.01\mu) \quad (18)$$

2) Possible Anderson localization effect due to the one dimensional stripes

At sufficiently low temperatures One dimensional conductors exhibit Anderson localization. Since we may wish to perform the experiments at low temperatures one may wonder if this can effect the striped plate conductivity and hence the proposed experiment. To show that these difficulties do not affect the proposed experiment we invoke again the discussion above of the relevant $J_\mu(x)$ configuration contributing to the casimir effect. In terms of the Fourier transform variables these are $\tilde{J}_\mu(k)$ with $k = 1/a$. Hence we do not need the stripes to be conducting along the full length L of the macroscopic sample. Rather it suffices that the stripes will be good conductors along distances l of order several times a . Since the distance between stripes d and the width of individual stripes fd and thickness t are also of order a (or rather smaller by a factor of 10 -100) the required conductivity of the stripes is practically guarenteed.

3)Possible effects from changing overlap of stripes (or other pattern). A basic requirement in our proposed set-up is that the separation between the plates a be larger than the separation/width of the stripes. In this case the modes relevant for the Casimir effect of wavelength $\approx 2a$ cannot resolve individual stripes. Hence $W(Cas)$ will not depend in this case on the relative positioning of the stripes in the two plates but only on the angle between their directions γ , which is exactly the effect of interest. The decreasing (in absolute value) of $W(\gamma)$ with γ and the resulting tendency of the plates to align their stripes (or whatever other common pattern of conducting patches exist on them), is a generic feature existing also when, d , the characteristic stripe (patch) size is larger than a . The new angle γ dependence follows from the fact that in this limit the total casimir energy is proportional to the total overlap area $A(overlap) = gfA$ with $A = L^2$ the complete plate area, f the fraction of conducting stripes/patches on each conductor and g is the the" overlap factor". If the patterns on both plates are identical (so that for $\gamma = 0$, say, $g = 1$) the overlap factor decreases with

increasing γ and so should $W_{Cas}(\gamma)$ for $d > a$. It is amusing to note that the change of $W(\gamma)$ in this case is qualitatively different from that in the $a > d$ the case of interest here, where as indicated in fig. $W(\gamma)$ varies smoothly as γ , the angle between the directions of conductivity changes from 0° to 90° . As one can readily verify the overlap of identical elements of size d (and also stripes of separation and width of order d) varies dramatically: decreasing from $g(\gamma = 0) = 1$ to its minimal value once $\gamma \approx d/L$. In passing we note that this dramatic sensitivity⁷ could conceivably be utilized for yet another measurement of the ordinary Casimir effect. The resulting torque in this case is $\frac{\Delta W}{\Delta \gamma} = \frac{\hbar c \pi^2 L^3 f}{720 a^4}$ and is comparable to the torques manifesting in the torque balance measurements of the ordinary Casimir effect.

Summing up we see then no obvious barrier to the proposed experiment. The feature of having angular dependence rather than only a dependent forces is rather welcome. Even if we use very thin coating stripes (of width $t = 0.01\mu$ which may be technologically advantageous, as no high, unstable grooves are required, the finite conductivity corrections only mildly influence the angular dependence of $F_{Cas}(\gamma, a)$. In this case the first $n = 1$ term in (12) will dominate the sum for E/A even more strongly since higher k values are cut off. While the force itself may be suppressed in proportion to t/t_{crit} with t_{crit} given by saturation of the bound in eq. (17), the angular dependence will be completely predictable and measurable.

Acknowledgment

We enjoyed discussions with E. Ben Jacob, G. Deutcher, J. Feinberg, R. Jaffe, E. Kapon, A. Mann, R. Onofrio, M. Revzen, and A. Palevski.

⁷This remarkable sensitivity to small rotations and also displacements of identical patterns is at the core of a methods for verifying such patterns, e.g., in the case of finger-print recognition [13].

References

- [1] S.Nussinov, “Polarized/Magnetic Version of the Casimir forces”, hep-ph/9509228(September 1995)
- [2] P.W.Miloni, R.J.Cook and M.F.Goggin, Phys.Rev A38, 1621(1988)
- [3] H.B.G. Casimir, “*on the attraction between two perfectly conducting plates*” Proc. K.Ned. Akad.Wet. 51, 793-796(1948)
- [4] O.Kenneth and S.Nussinov, preprint hep-th/9802149.
- [5] O.Kenneth thesis Tel-Aviv University 1999
- [6] H.Li and M.Kardar, Phys.Rev.Lett.**67**,3275(1991);Phys.Rev.A46,6490(1992).
- [7] R. Golestanian, M.Kardar quant-ph/9802017
- [8] O. Kenneth,S.Nussinov*Small object limit of Casimir effect and signs of the Casimir force*(to appear next week)
- [9] S.K.Lamoreaux, “*Demonstration of the Casimir force in the 0.6 to 6 μ m Range*”,Phys Rev. Lett. 78(1) 5-8(1997)
- [10] U.Mohideen, and Anushree Roy. preprint physics/9805038
Anushree Roy,Chiung-Yuan Lin and U.Mohideen quant-ph/9906062
Phys. Rev. D60:111101(1999)
- [11] Sparnaay.M.J.,1957, Nature,180,334;1958,Physica,24,751;
- [12] J.Schwinger,L.L.DeRaad,K.A.Milton,Ann. Phys.(N.Y)115,1(1978)
- [13] Joseph.Johnson. private communication